Exercise 2

Show that

$$\frac{1}{1/z} = z \qquad (z \neq 0).$$

Solution

$$\begin{aligned}
\frac{1}{1/z} &= \frac{1}{\frac{1}{x+iy}} \\
&= \frac{1}{\frac{1}{x+iy} \cdot \frac{x-iy}{x-iy}} \\
&= \frac{1}{\frac{x-iy}{x^2-i^2y^2}} \\
&= \frac{1}{\frac{x-iy}{x^2+y^2}} \\
&= \frac{1}{\frac{1}{x^2+y^2}} \cdot \frac{1}{x-iy} \\
&= (x^2+y^2) \cdot \frac{1}{x-iy} \cdot \frac{x+iy}{x+iy} \\
&= (x^2+y^2) \cdot \frac{x+iy}{x^2-i^2y^2} \\
&= (x^2+y^2) \cdot \frac{x+iy}{x^2+y^2} \\
&= x+iy \\
&= z
\end{aligned}$$